Cognitive bias approach to the acquisition of disjunction[1]
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Abstract. While most adults tend to interpret disjunctive statements like '*A or B*' exclusively, some children (age 3-5/6) have a conjunctive interpretation, and some an inclusive interpretation. We propose that the behaviour of the conjunctive children can be explained by the lack of two abilities: 1. The ability to engage with models that verify sentences by virtue of an empty witness set. 2. The ability to entertain two alternative ways the world can be in parallel. We argue against theories explaining this behaviour in terms of alternative-based reasoning. In particular, our theory predicts that the order of acquisition of the meaning of disjunction starts with the conjunctive interpretation, followed by inclusive and exclusive interpretations, while alternative-based theories predict that children acquire the inclusive reading first. We relate our proposal to the acquisition of modal concepts.

Keywords: disjunction acquisition, neglect-zero, exhaustification, modal cognition.

1 Introduction

Classically, disjunction (*or*) has two interpretations: the first one (inclusive) evaluates disjunction as true if at least one of the two disjuncts holds, as in (1a), and the second if exactly one disjunct is true, as in (1b). Across languages, adults usually interpret disjunction exclusively, while the inclusive interpretation remains possible (Nicolae et al., 2024). Surprisingly, many studies report that children (age 3-5/6) in various languages interpret disjunction conjunctively, i.e., they only evaluate it as true if *both* disjuncts are true, as in (1c)[2]. These interpretations of a disjunctive sentence are represented in Figure 1.

(1) Ann ate an apple or a banana.
   a. Ann ate at least one of the two pieces of fruit. (Inclusive: $\alpha \vee \beta$)
   b. Ann ate exactly one of the two pieces of fruit. (Exclusive: $(\alpha \vee \beta) \wedge \neg(\alpha \wedge \beta)$)
   c. Ann ate both an apple and a banana. (Conjunctive: $\alpha \wedge \beta$)

In this paper, we present a novel hypothesis concerning the source of conjunctive interpretations. We propose that the behaviour of conjunctive children is due to two related cognitive biases: *neglect-zero* and *no-split*. Conjunctive children have trouble engaging with *zero-models*, i.e., models that verify a sentence by virtue of an empty configuration, and lack the ability to *split*, i.e., to process (possibly conflicting) alternative possibilities in a parallel fashion while interpreting one sentence. On this hypothesis, conjunctive interpretations are acquired earlier than inclusive or exclusive ones (cf. Singh et al. 2016) and, as predicted by the logical modelling of

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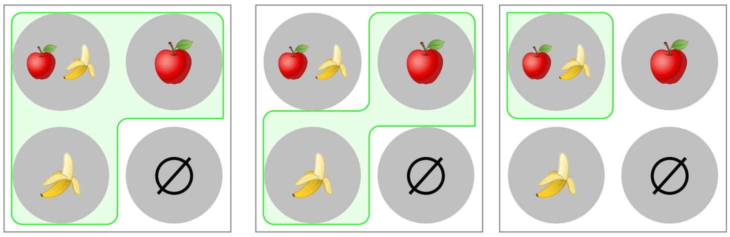
[2]See e.g., studies by Singh et al. (2016), Tieu et al. (2017), Cochard et al. (2023), Bleotu et al. (2024a) a.o.



(a) Inclusive (b) Exclusive (c) Conjunctive

Figure 1: Interpretations of disjunction

our two biases, they persist under e.g., deontic modals and universal quantifiers, but not under negation (Cochard et al., 2023, 2024; Pagliarini et al., 2018).

In the next section, we will describe the phenomenon in more detail and present its existing explanations. In Section 3, we will discuss the working of the two postulated biases and show how the existence of the biases explains the behaviour of conjunctive children. Moreover, we will offer evidence of the cognitive reality of the biases from adult language and children's behaviour in other tasks. In Section 4, we will compare our proposal to an alternative-based approach proposed by Singh et al. (2016), and identify predictions that allow to experimentally distinguish between the theories.

2 Conjunctive readings of disjunction

English-speaking children start producing *or* around the age of 2;6, and, by the age of 3;8, the production frequency reaches one *or* per 1000 words. They learn to use *and* much earlier and use it much more frequently. They can productively use *or*, but they still differ from adults (their parents) in both production frequency (adults produce 1.5 *or* per 1000 words) and use (Morris, 2008; Jasbi et al., 2024). Moreover, in comprehension tasks, children exhibit non-adult-like patterns. In particular, while adults across languages tend to interpret disjunction exclusively, i.e., when only one of the disjuncts is true, children are happy to accept *or*-sentences as true in the case where both disjuncts are true (see e.g., Paris (1973); Braine and Rumain (1981); Chierchia et al. (2001); Gualmini et al. (2001)).

Since the exclusive interpretation of disjunction is postulated to follow from alternative-based reasoning, Chierchia et al. (2001) and Noveck (2001) proposed that the ability to perform such reasoning is developed late, and that children are in some sense "perfect logicians" – they only use the literal unstrengthened interpretations of *or*. They confirmed this hypothesis by showing that children did not strengthen the interpretation of other expressions requiring alternative-based reasoning: Sentences with *some* were accepted in cases where *all* objects satisfied the property, and sentences with *might* in cases where *has to* would be more accurate.

Interestingly, numerous studies indicate that children (3-5/6 years old) not only accept disjunctive sentences in the case where both disjuncts are true, but also reject them in the canonical cases, where exactly one disjunct is true. Hence, they have a conjunctive interpretation of disjunction. Singh et al. (2016) observed that some children interpret unembedded disjunctions as conjunctions (2), and that this also persists under a universal quantifier (3).[3] Cochard et al. (2024) observed it under an existential modal (4). Further research by Tieu et al. (2017) indicates that both French- and Japanese-speaking children interpret both simple and complex disjunctions as either inclusive or conjunctive; in contrast, adults generally accessed the exclusive readings. Similar results were found for French by Cochard et al. (2023), for Romanian by Bleotu et al. (2024a), and for Mandarin Chinese by Yan and Luo (2025). [4]

(2) Ann ate an apple or a banana. $\alpha \vee \beta$
$\rightsquigarrow$ Ann ate both an apple **and** a banana. $\alpha \wedge \beta$

(3) Every girl ate an apple or a banana. $\forall x(Px \vee Qx)$
$\rightsquigarrow$ Every girl ate an apple **and** a banana. $\forall x(Px \wedge Qx)$

(4) Ann may eat an apple or a banana. $\Diamond(\alpha \vee \beta)$
$\rightsquigarrow$ Ann may eat both an apple **and** a banana. $\Diamond(\alpha \wedge \beta)$

In the remainder of this section, we will discuss the existing approaches to explaining the phenomenon of conjunctive children. First, we will address two worries, which challenge the reality of the phenomenon, and then the alternative-based approach.

2.1 The null hypothesis

The first issue we should address is whether the effect we discuss is real; whether children really have conjunctive interpretations of disjunction. There are two possible objections going in opposite directions against the claim that the data needs further explanation:

1. The children are genuinely confused about the meaning of disjunction, and they identify it with conjunction, as it plays the same syntactic role (lexical misanalysis).

2. The data is an experimental artifact, and children are (in various circumstances) capable of interpreting disjunction correctly (Skordos et al., 2020; Huang and Crain, 2020).

2.1.1 The hypothesis of lexical misanalysis

The first objection is addressed in the existing literature by Singh et al. (2016), who observe that children distinguish the two connectives in downward entailing environments: under negation or in the antecedents of conditionals. This is confirmed by Notley et al. (2016) as well as Pagliarini et al. (2018), who observed that children in various languages interpret negated disjunctions, as *neither...nor* (5). This is somewhat surprising because the plain negation of a conjunction would produce a weaker reading (*not both*). Tieu et al. (2017) pointed out that Su (2014) provides additional evidence that they are also distinguished by Mandarin-speaking children in the consequents of conditionals.[5]

[3] Pagliarini et al. (2018) observed that the inference is less frequent under a universal quantifier.

[4] It is important to mention that Skordos et al. (2020) and Huang and Crain (2020) argued against the claim that children have a truly conjunctive interpretation of disjunction. They claim that the results by Singh et al. (2016) and Tieu et al. (2017) are due to an experimental artifact. However, the studies of Cochard et al. (2023, 2024) and Bleotu et al. (2024a) show that conjunctive reading persists even when the confounding factors are removed. We will engage with this discussion in Section 2.1.2.

[5] Even though, Su (2014) provides direct comparison of disjunction and conjunction in the same position, we may not be certain that any children in the study were in fact conjunctive. However, the study reveals big differences in

(5) Ann did not eat apples or bananas. $\neg(\alpha \vee \beta)$
$\rightsquigarrow$ Ann did not eat apples and she did not eat bananas. $\neg\alpha \wedge \neg\beta$

Pagliarini et al. (2018) reviewed the literature on the interpretation of negated conjunctions and disjunction in several languages. For adults, the interpretations differ from language to language, e.g., in some languages like Italian, disjunction is a positive polarity item (PPI) and takes wide scope over negation (Szabolcsi, 2004). However, if we focus on children, we observe that the interpretations are the same across languages (see Table 1 for summary). In all considered languages, children interpret the negation of disjunction classically, as in (5) – hence they do not treat it as a conjunction. However, children also interpret negated conjunctions as *neither...nor*, which collapses the truth conditions of disjunction and conjunction under negation as in (6). Hence, we cannot simply take the success with disjunction under negation to be evidence against the lexical misanalysis hypothesis. One could argue in its defense that children systematically interpret both *or* and *and* as *wide-scope* conjunction, which would derive the readings (2), (3), (5) and (6). However, this hypothesis would fail to explain (4) since $\Diamond(\alpha \wedge \beta) \not\equiv \Diamond\alpha \wedge \Diamond\beta$.

(6) Ann did not eat apples and bananas. $\neg(\alpha \wedge \beta)$
$\rightsquigarrow$ Ann did not eat apples and she did not eat bananas. $\neg\alpha \wedge \neg\beta$

Language	NOT OR	NOT AND	Paper
English	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	Notley et al. (2016)
Japanese	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	Goro (2007)
Mandarin	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	Crain (2012); Notley et al. (2016)
Turkish	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	Goro (2007); Geçkin et al. (2018)
Italian	$\neg p \wedge \neg q$	$\neg p \wedge \neg q$	Goro (2007); Geçkin et al. (2018)

Table 1: Children's interpretation of negated conjunction and disjunction.

Hence, to defend the hypothesis of lexical misanalysis, one has to maintain that children think of *or* and *and* as one connective which always receives the strongest interpretation: a conjunctive interpretation in the positive and a disjunctive interpretation in the negative. Clearly such a connective does not correspond to either of the classical $\wedge$ and $\vee$. This claim makes the hypothesis much more complex, as it would still require a semantic (or cognitive) explanation of why children always opt for the strongest interpretation. Both the cognitive-bias approach and the alternative-based approaches have answers to this question.[6]

Additional evidence against the claim that the children are simply confusing the two connectives comes from the work by Bleotu et al. (2024b), who show that children can be forced to access the

children's interpretation of the sentences: children compute (i), only for disjunction and (ii), only for conjunction; their reasoning satisfies Simplification of Disjunctive Antecedents, but they do not simplify conjunctive ones. Similarly, they do not strengthen the disjunctive consequents but keep the strong meaning of conjunction. (i) is consistent with the claim that children take disjunction to be a conjunction but take wide scope over the conditional. This, however, does not work for (ii) since the only way to block the inference is by accessing the disjunctive meaning. Similar points can be raised for other environments in which children assign the correct interpretation to disjunction (see the discussion by Tieu et al., 2016).

(i) If Ann conjures up a rabbit or a bear, she can get a reward. $(\alpha \vee \beta) \rightarrow \gamma$
$\rightsquigarrow$ If Ann conjures up a rabbit, she can get a reward. $\alpha \rightarrow \gamma$

(ii) If Ann conjures up a rabbit, she will get a ball or a star. $\alpha \rightarrow (\beta \vee \gamma)$
$\not\rightsquigarrow$ If Ann conjures up a rabbit, she will get a ball and a star. $\alpha \rightarrow (\beta \wedge \gamma)$

[6]See Section 3.2 of this paper and e.g. Sbardolini (ms) and Bar-Lev (2021).

disjunctive meaning when evaluating incompatible disjuncts. In particular, the children do not derive conjunctive conclusions if they are contradictory, as in (7). Instead, they are much more likely to accept the disjunction in the case where exactly one disjunct is true (it was accepted 89% of the times). This behaviour is significantly different from the conjunctive control, where children accepted the conjunction in the same context (one conjunct true) only 14% of the time.

(7) The squirrel is at the top or at the bottom of the tree.
$\not\rightsquigarrow$ The squirrel is at the top *and* at the bottom of the tree.

The result by Bleotu et al. (2024b) shows that conjunctive children are able to access the inclusive interpretation of disjunction when forced to do so in case their reasoning ends up contradictory. This strategy is not available with an explicit conjunction present in place of a disjunction, which suggests that they are able to distinguish between the two connectives even in an unembedded environment, against the prediction of the lexical misanalysis hypothesis.

2.1.2 The experimental artifact approach

Skordos et al. (2020) replicated the results by Tieu et al. (2017), and provided new conditions with two critical changes, which resulted in the number of children interpreting conjunction conjunctively dropping from 14 in the replication to 4 in the changed version of the experiment. The crucial change concerns increasing the number of alternatives available in the stimuli. Skordos et al. (2020) claim that when only two alternatives are available, as in the original experiment, an utterance of disjunction is infelicitous since it is just a vacuous guess ("Just as it is odd to predict the outcome of a tennis match by saying, *Either A or B will win*."), as it trivializes the Question Under Discussion. Similar critique was raised by Huang and Crain (2020).

However, recent experiments show that some children interpret some disjunctions (e.g., *fie...fie* in Romanian) conjunctively even if four alternatives are present. Moreover, we see from the research by Singh et al. (2016), Pagliarini et al. (2018), and Cochard et al. (2024) that some children still interpret the disjunction conjunctively in environments where the set of alternatives does not seem to trivialize the disjunctive answer to the QUD (e.g., under modals or quantifiers).

From the presented empirical picture, we can conclude that some children interpret (some) disjunctions as conjunctions and that this phenomenon still requires an explanation. Moreover, we conclude following Skordos et al. (2020) that the change in interpretation in various environments and context requires further empirical investigation.

2.2 The alternative-based approach

In conversations, sentences can be strengthened with an implicature (Grice, 1975). Strengthening happens via negating alternatives to the sentence (Horn, 1972). The exclusive reading of disjunction is assumed to follow exactly from alternative-based reasoning as in (8), where the conversational meaning of *or* includes the negation of the stronger alternative – *and* (see e.g., Sauerland, 2004; Spector, 2007a, b).

(8) Ann ate an apple or a banana.
ALT: Ann ate both an apple and a banana.
$\rightsquigarrow$ Ann did not eat both an apple and a banana.

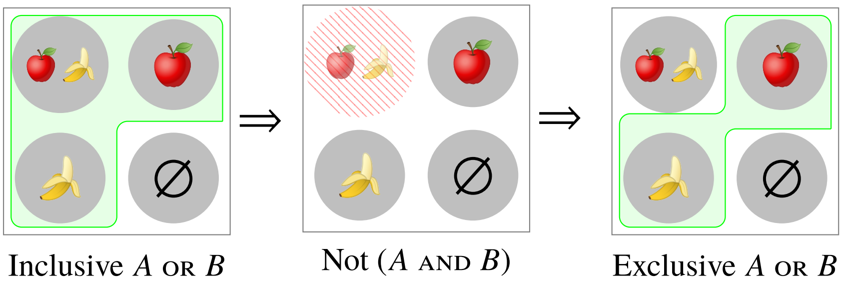


Figure 2: Derivation of the exclusive interpretation in an alternative-based view.

In grammatical approaches to scalar implicatures, the modeling of alternative-based reasoning is based on the *exh* operator, which relies on a syntactic computation of the alternative set and semantic derivation of the excluded alternatives. This operator can be iterated to allow deriving more complex inference patterns such as Free Choice inferences (Fox, 2007; Bar-Lev, 2018).

Since Noveck (2001) and Chierchia et al. (2001) a common assumption was that the ability to exhaustify sets of alternatives is acquired late and that young children (3-5/6 years old) do not negate stronger alternatives when interpreting weak claims. This was based on the observation that from *some* they do not conclude *not all* and from *or* they do not conclude *not and* (as explained in the introduction). This observation led to the conclusion that children are unable to apply even one *exh* operator.

However, the data by Singh et al. (2016) and Tieu et al. (2017) challenge this conclusion. It is not the case that (at least when it comes to disjunction) children are more logical than adults, as claimed in the title of (Noveck, 2001), but that they simply assign incorrect interpretations. Singh et al. (2016) assume instead, following Sauerland (2004) and Katzir (2007)'s algorithms for generating alternatives grammatically, that there are two sources of alternatives (for adults):

1. Replace nodes with their sub-constituents or other salient constituents.
2. Replace terminals with other lexical items.

Hence in a disjunctive statement $p \vee q$ the first source allow adults to access the disjuncts separately ($\{p, q\}$) and the second source to replace disjunction with conjunction ($\{p \wedge q\}$). Thus, the set of alternatives of such a sentence is the sum of the outcomes of those two operations: $Alt_a(p \vee q) = \{p \vee q, p, q, p \wedge q\}$. The computation of alternatives for a sentence like $\exists xPx$ only relies on the second mechanism, since they are terminal (do not have proper subconstituents). Thus $Alt_a(\exists xPx) = \{\exists xPx, \forall xPx\}$. The interpretation of a sentence is calculated by exhaustifying the set of its alternatives following Fox (2007) or the more recent Bar-Lev (2018).

Singh et al. (2016) argue that children's interpretations of sentences are different from adult not because of a different mechanism (or lack thereof) but because of a different set of alternatives children work with. They argue that children acquire the ability of alternative-based reasoning early, but they have trouble learning the process of generating sets of alternatives: They do not have the ability to access the lexicon in search for alternatives, and only derive them via subconstituents. Hence $Alt_c(p \vee q) = \{p \vee q, p, q, \cancel{p \wedge q}\}$ and $Alt_c(\exists xPx) = \{\exists xPx, \cancel{\forall xPx}\}$.

Using this set of alternatives, the conjunctive reading can be derived via the following procedure and using Fox (2007)'s definition of *exh* for $exh(exh(p \vee q))$:

Step 1 Inclusive OR: Ann ate an apple or a banana.

Step 2 *ALT:* 1. Ann ate an apple. 2. Ann ate a banana.

Step 3 Mutual negation of alternatives:
ALT_{exh}: 1. Ann ate *only* an apple. 2. Ann ate *only* a banana.

Step 4 Negation of the exhaustified alternatives:
$\rightsquigarrow$ 1. Ann did **not** *only* eat an apple. 2. Ann did **not** *only* eat a banana.

Step 5 Ann ate **both** an apple and a banana.

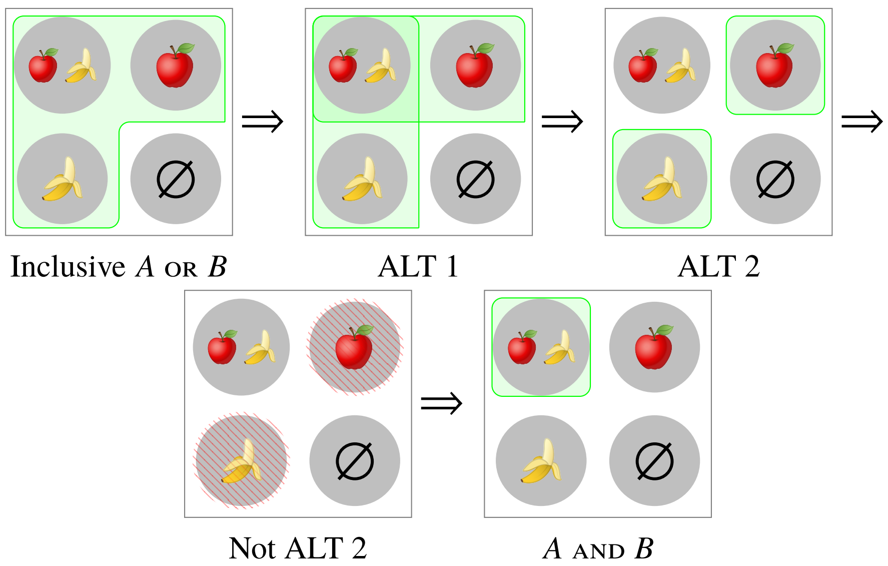


Figure 3: Illustration of the computation of the conjunctive meaning of disjunction in alternative-based approaches.

Singh et al. (2016) claim that children *are* capable of performing recursive exhaustification,[7] but perform it on an incorrect set of alternatives, which leads them to accessing the conjunctive parse. The preference for a conjunctive strengthening as opposed to choosing the inclusive parse follows from a pragmatic preference for a complete answer to the QUD.

What remains to be explained is why children still often interpret disjunctions inclusively. There are at least two reasons why this may be the case. 1. They might be unable to access any alternatives - when they cannot even access the subsonstituents as alternatives (Singh et al., 2016). 2. They might not have learned yet to perform the alternative-based reasoning (Tieu et al., 2017; Cochard et al., 2023). Both of these options have one assumption in common: children

[7] Bar-Lev and Fox (2020) observe that the same result can be achieved if we assume that children are able to perform the Innocent Exclusion and Inclusion algorithm:

1. Take the prejacent and compute the set of alternatives $Alt_c(p \vee q) = \{p \vee q, p, q, \cancel{p \wedge q}\}$
2. Take all maximal sets of alternatives that can be assigned *false* with the prejacent: $\{p\}, \{q\}$
3. *Innocent Exclusion:* Exclude (assign false) the intersection of those sets: $\emptyset$
4. Take all maximal sets of alternatives, that can be assigned *true* with the prejacent and negations of excluded alternatives. $\{p \vee q, p, q\}$
5. *Innocent Inclusion:* Include (assign true) the intersection of those sets. Thus if for all $\varphi \in \{p \vee q, p, q\}$ we know that $V(\varphi) = 1$, so also that $V(p \wedge q) = 1$.

must be able to compute the inclusive parse *before* (or at least no later than) they compute the conjunctive parse.[8] Thus, in this theory, the order of acquisition is predicted to be as follows:

$$\textsc{inclusive}\ (1a) \precsim \textsc{conjunctive}\ (1c) \prec \textsc{exclusive}\ (1b)$$

This is the key difference in predictions between the account proposed in this paper and the exhaustification-based accounts. We will discuss the predictions of this account in Section 3.2. Singh et al. (2016) seem to propose that children know the meaning of disjunction (the inclusive parse), but use it only to go through a complicated process to arrive at an incorrect conjunctive reading. Moreover, children systematically and across languages choose that meaning over the inclusive reading. While this story derives part of the behavior of the conjunctive children, we find it cognitively unrealistic, as it attempts to explain children's behavior purely in terms of lack of lexical knowledge. Instead, we would like to propose a theory that points out limitations in children's cognitive capacities that lead to the conjunctive interpretations. We will present this theory in the next section.

3 The cognitive bias approach

When interpreting a sentence, speakers create structures representing reality (e.g., Johnson-Laird (1983)'s mental models). We argue, following Aloni (2022), that some strengthened interpretations of sentences follow from speakers neglecting certain (cognitively complex) representations from their models of reality. Such simplifications (biases), can cause mistakes or irrational behaviour. We hypothesize that the occurrence of biases and the use of heuristics (at least in the particular case of natural language strengthening discussed in this paper) follow from neglecting cognitively complex representations in the interpretation process. In that sense, heuristics can be a case of logical/rational reasoning, but are based on a simplified structure used to represent reality.[9]

3.1 Neglect-zero

Aloni (2022) argued that people tend to neglect structures that verify a sentence by virtue of an empty configuration (*zero-models*). This tendency is independently motivated by the attested difficulty of the cognitive operation of evaluating (true) sentences with respect to empty witness sets (Nieder, 2016; Bott et al., 2019). And has been empirically confirmed by natural language studies (see e.g., Degano et al., 2025; Bott et al., 2025; Klochowicz et al., 2025).

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[8]Singh et al. (2016) propose that children access the conjunctive interpretation in the choice between parses. Hence, a third explanation of the inclusive behavior of children is available: children may get the classical inclusive interpretation in case they (for some reason) choose the inclusive parse instead of the conjunctive one. This third option allows for the possibility that the two parses are acquired at the same time: if recursive exhaustification is available to them before they learn any meaning of disjunction, they may directly exhaustify the acquired inclusive meaning and access both parses. Moreover, this explanation is consistent with children who first interpret disjunctions conjunctively and only with time inclusively. For instance, if the parse-choice criterion shifts from preferring complete answers to QUDs to preferring simpler expressions (without exhaustification).

[9]Simon (1955) proposed to think of rationality as bounded by computational constraints, and Kahneman et al. (1982) observed that human reasoning is systematically affected by biases. Others observed that we access strengthened interpretations as resulting from biases or heuristics (see, e.g. Williamson, 2024). However, we do not commit to the details or broader implications of these theories, such as, for example, Kahneman et al.'s two systems or the particular construction of Johnson-Laird's mental models.

Let's illustrate the working of the bias with an example. Consider the sentence in (9) and some example models of it. When interpreting sentences like (9), people tend to consider models listed as *Verifiers*, since they both make the sentence literally true and contain some mentioned objects (i.e., black squares). However, (9) is also true in the case of *zero-model verifiers* which contain no black squares (since 0 < 3). However, people judge sentences like (9) as false 30-50 % of the time in cases where there are no black squares, i.e., when the scope is empty. (Bott et al., 2019, 2025; Klochowicz et al., 2025). This happens because in these cases the sentence is verified by an *empty set* of black squares. Aloni (2022) postulates that representing such empty configurations is cognitively taxing and, by default, does not occur in the representation process.

(9) Less than three squares are black.
 a. Verifiers: [■, □, ■]; [■, □, □]
 b. Falsifier: [■, ■, ■]
 c. Zero-model verifiers: [□, □, □] ;[▲, ▲, ▲]; [□, □, ▲].

Another inference, which, according to Aloni (2022), follows from the neglect-zero tendency, is the ignorance inference of disjunction (10).[10] Grice (1989) observed that asserting a disjunction does not merely require (exactly) one of the disjuncts to be true, but also the *possibility* of each disjunct to be true. [11] The possibility here is defined broadly, i.e., in (10a) it could be speaker-oriented (i.e., the speaker thinks that the prize can be in either place) or listener-oriented (the speaker knows where the prize is, but does not want to tell the listener exactly, hence they imply that there are two possibilities). In particular, (10b) is odd in the speaker-oriented possibility case, as it implies that the speaker doesn't know how many children they have.

(10) a. The prize is in the attic *or* in the garden. $\alpha \vee \beta$
 ⇝ It might be in the attic, and it might be in the garden. $\Diamond\alpha \wedge \Diamond\beta$
 b. ??I have two *or* three children.
 ⇝ the speaker doesn't know how many children they have.

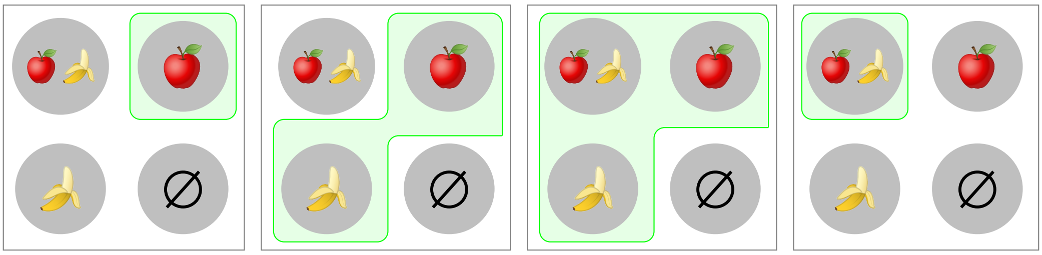
Aloni (2022) explains the inference in (10) in terms of the neglect-zero tendency. She argues that for a disjunction to be assertable, both disjuncts need a (non-empty) witness set of possibilities. For instance, in Figure 4, the sentence (11) is verified emptily by the 4(a), since in this case, it is not possible for the second disjunct (eating a banana) to be true, as in this case, Ann ate the apple and not the banana. The other verifiers (4(b), 4(c), and 4(d)) include eating a banana as a possibility and hence verify the disjunction non-emptily.

(11) Ann ate an apple or a banana.
 ⇝ The speaker does not know which fruit she ate.

Aloni (2022) postulates that conceiving the empty verifier for a sentence like (11) is cognitively taxing, and by default does not happen. Hence, adults draw the ignorance inference since in all

[10]Neglect-zero bias can account for a range of phenomena involving disjunction, e.g., *free choice* and *distributive inferences*. We will not discuss these applications here.

[11]Sauerland (2004), Fox (2007) and others framed this inference in terms of *uncertainty* about each disjunct being true and derived the possibility inference from uncertainty via alternative-based reasoning. Degano et al. (2025) show that this derivation is not attested empirically, since participants draw the possibility inference even in cases where there is no uncertainty about one of the disjuncts. Hence, we follow Degano et al. (2025) here in assuming that the ignorance inference is primarily a possibility inference.



(a) Empty verifier (b) Non-empty verifier (c) Non-empty verifier (d) Non-empty and non-split verifier

Figure 4: Verifiers of disjunction (11).

representations of the sentence, both disjuncts are possible.

3.2 No-split

Observe that the non-empty verifiers for disjunctions involve representing *multiple possibilities* while evaluating one sentence. For instance, 4(b) requires entertaining the possibility where only the apple was eaten and another one, where only the banana was eaten. Hence 4(b) and 4(c) are *split* verifiers: they require splitting the possibility space into parts and evaluating the truth of the sentence with respect to each part while keeping the others in working memory.

Following Sbardolini (ms), we argue that *split* verifiers are also cognitively taxing, and that under certain conditions people (especially children) can fail to use them and collapse the multiplicity of possibilities into one.[12] In that case, the only verifiers considered are the non-zero and no-split verifiers such as 4(d). Observe that 4(d), which corresponds to a conjunction, is the only verifier for (11) which is both non-zero and non-split. Hence, under both biases, disjunctions are interpreted as conjunctions. We conjecture that the ability to split states is acquired late, and that the combination of the neglect-zero and the *no-split* biases can explain the non-classical inferences (conjunctive readings) observed in pre-school children.

Observe that the notion of *split* is independent of the particular formalisation of modal thought. In a standard possible-world semantics, possibilities correspond to accessible possible worlds. Since disjunction requires representing two possibilities separately, in that framework, *split* corresponds to the ability to consider two or more possible worlds in the image of the accessibility relation. Similar formalization of *split* applies to Aloni (2022), where *split* corresponds to evaluating formulas with respect to states consisting of two or more worlds, and to Sbardolini (ms) where *split* is the ability to entertain states with two or more proper parts.[13] In Phillips and Kratzer (2024)'s situation semantics split corresponds to considering two or more extensions

[12]For exposition purposes, we present the biases as distinct mechanisms. Sbardolini (ms) presents them as cumulative: including zero requires the ability to split. Whether or not the biases are independent, we think that it is not likely to not be able to split, but be able to reason with zero, as the cognitive complexity of empty representations is challenging even for adults.

[13]For instance, by neglect-zero the disjunction $p \vee q$ requires support of two possibilities: $\Diamond p$ and $\Diamond q$. In Aloni's framework, *split* will allow to evaluate at states that are not singleton, and hence can be split verifiers as exemplified above. In standard possible-world semantics, *split* will correspond to the ability to consider $R[w] = \{v_p, v_q\}$ and note merely $\{v_{pq}\}$, which entails the conjunction $\Box(p \wedge q)$. We briefly propose a formal framework in Section 3.4, but the working of the biases is independent of this particular proposal.

of the modal anchor (see more in Section 3.3). And in Johnson-Laird (1983)'s mental model theory it corresponds to engaging with two or more mental models at the same time.

Therefore, we predict that children are conjunctive if they have not yet fully acquired the ability to split, and hence they might not perform this operation. The inclusive reading should appear later, when they realize that splitting is necessary for the effective use of disjunction. This meaning can be further strengthened by means of alternative-based reasoning to the exclusive interpretation (1b), which could happen later as postulated by Chierchia et al. (2001) and Noveck (2001). Thus, we predict the following order of acquisition:

CONJUNCTIVE (1c) $\precsim$ INCLUSIVE (1a) $\precsim$ EXCLUSIVE (1b)

Above, we have seen that the neglect-zero bias is empirically attested in various experimental paradigms; zero models are indeed frequently neglected, and if included, they cause delay in processing. Below, we present some independent evidence for the cognitive reality of the *no-split* bias. [14] In particular, we will focus on children's difficulties with representing possibilities.

3.3 No-split and the development of modal cognition

A vast body of literature showed that 3- and 4- years old children have troubles with reasoning about possibilities (see e.g., Leahy and Carey (2020) or Phillips and Kratzer (2024) for a review). In particular, children have difficulties with considering two mutually inconsistent possibilities: in a Y-shaped (tube) task (Beck et al., 2006; Redshaw and Suddendorf, 2016; Redshaw et al., 2019; Leahy, 2024). In the first task, children are presented with a Y-shaped tube and a ball that is dropped from the top of the tube and can go either left or right, as in Figure 5. The task is to catch the ball. While adults and older children are able to figure out the optimal solution, i.e., cover both exits, younger children (2-, 3-, and 4- year olds, depending on the studied population) and great apes tend to cover one exit at random. This behaviour pattern indicates that, "[...] acknowledging multiple possibilities poses serious problems for them [3- and 4-year-olds]. By around 5 or 6 years, children can acknowledge multiple possibilities" (Beck et al., 2006: p. 425).

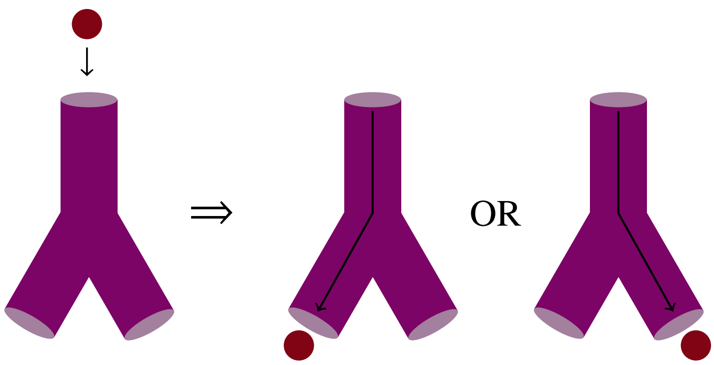


Figure 5: Y-shaped tube task by Redshaw and Suddendorf (2016), inspired by the design of Beck et al. (2006)'s experiment.

Observe that in order to find the correct solution to the Y-shaped tube, the participants need to imagine two mutually exclusive possibilities (ways the ball could go), and act on both of them

[14] We thank Jeannette Schaeffer for pointing out that such evidence would provide support for our theory.

simultaneously. Hence, in particular, if children have not acquired the ability to split, they will not be able to find the correct solution. A conjunctive child will not conclude that the ball will go *left and right*, because object permanence (persistence) is usually acquired within the first year of life (Piaget and Cook, 1954; Bremner et al., 2015). To find a solution, the child can either perform the *split*, which is cognitively taxing but forced by the contradictoriness of the conjunctive reading, or choose one disjunct at random and hope for the best. Observe that, in particular, children's behavior in the Y-shaped tube task is analogous to behavior in the linguistic experiments about mutually incompatible disjuncts like (7) by Bleotu et al. (2024b).

We agree with Phillips and Kratzer (2024) that children's failure in the Y-shaped tube tasks does not have to be due to a lack of concept of possibility *per se* (cf. Leahy and Carey (2020)). Even if children can reason about (future) possibilities, they may still have difficulties with reasoning about two or more possibilities simultaneously, i.e., with *split*. It is important to notice that the age at which children start solving the Y-shape tube task correctly coincides with the age where conjunctive readings tend to disappear.

To explain children's behavior, Leahy and Carey (2020) postulate the *minimal representation* theory. They argue that children simulate a single outcome (possibility) and act based on this simulation (cf. Leahy (2024)). This does not seem to extend to the linguistic behaviour of conjunctive children. We argue that the possibilities that children engage with do not have to be minimal, as in the linguistic tasks, children pick a conjunctive interpretation and not a single-conjunct interpretation, but that they are bound by complexity: the representations do not involve entertaining multiple possibilities simultaneously. In the cases of mutually incompatible possibilities in linguistic and physical tasks (like the Y-shaped tube), the two approaches coincide, as the conjunctive interpretation is ruled out for independent reasons.

3.4 The formal treatment of the biases

In this paper, we will only hint at the formalism that can be used to model the biases. We propose to treat the biases as modal-theoretical restrictions on the complexity of the information structure available to the reasoners (in this case, children) following Aloni (2022)'s BSML*. Let $M = (W, R, V)$ be a Kripke model and let $S \subseteq \mathcal{P}(W)$ be the set of information states available to a reasoner. The logic below has the same consequence relation as classical logic when $S = \mathcal{P}(W)$, but once we restrict S, the logic becomes non-classical and allows us to make predictions about inference patterns under cognitive bias. Hence, biases will be modeled as complexity-based restrictions on S. Let's assume that if $s \notin S$ then for any φ: $M, s \not\models \varphi$ and $M, s \not\dashv \varphi$. Otherwise:

$M, s \models p$ iff for all $w \in s : V(w, p) = 1$
$M, s \models \neg\varphi$ iff $M, s \dashv \varphi$.
$M, s \models \varphi \vee \psi$ iff there are $t, t' : t \cup t' = s$ and $M, t \models \varphi$ and $M, t' \models \psi$.
$M, s \models \varphi \wedge \psi$ iff $M, s \models \varphi$ and $M, s \models \psi$.
$M, s \models \Diamond\varphi$ iff for all $w \in s$ $\exists t \subseteq R[w] : t \neq \emptyset$ and $M, t \models \varphi$.

$M, s \dashv p$ iff for all $w \in s : V(w, p) = 0$
$M, s \dashv \neg\varphi$ iff $M, s \models \varphi$.
$M, s \dashv \varphi \vee \psi$ iff $M, s \dashv \varphi$ and $M, s \dashv \psi$.
$M, s \dashv \varphi \wedge \psi$ iff there are $t, t' : t \cup t' = s$ and $M, t \dashv \varphi$ and $M, t' \dashv \psi$.
$M, s \dashv \Diamond\varphi$ iff for all $w \in s$: if $t \in S$ and $t \subseteq R[w]$ then there is a t': $t' \in S$ & $t \subseteq t' \subseteq R[w]$ & $M, t' \dashv \varphi$.

Entailment is be defined classically, i.e., $\varphi \models \psi$ iff for any s: if $M, s \models \varphi$ then $M, s \models \psi$. This framework can be extended to a First Order version following Aloni and van Ormondt (2023). As discussed in Section 3.1, adults tend to neglect structures that verify a sentence by virtue of an empty configuration (zero-models). In the current framework, this bias can be modeled by postulating $S = \mathcal{P}(W) \setminus \varnothing$; by excluding the empty set from available information states. The no-split bias can be modeled as allowing only states, which cannot be split into proper parts: as $S = \{t \mid |t| \leq 1 \,\&\, t \subseteq W\}$ [15]. Both biases put a ban on more complex states; while the complexity of the empty state follows from the qualitative difference of empty representations, the no-split bans states whose increased complexity corresponds to larger cardinality. In particular, observe that if $S = \{t \mid |t| \leq 1 \,\&\, t \subseteq W\} \setminus \varnothing$, which is the result of applying both biases, then $\varphi \vee \psi \models \varphi \wedge \psi$; $\neg(\varphi \wedge \psi) \models \neg\varphi \wedge \neg\psi$ and $\Diamond(\varphi \vee \psi) \models \Diamond(\varphi \wedge \psi)$ all hold (but still $\Diamond\varphi \not\models \Box\varphi$).

4 Evaluation of the theories

In this section, we will compare the predictions of the alternative-based account by Singh et al. (2016) and the current proposal to provide a more complete theoretical and empirical picture. Since Singh et al. (2016)'s theory is about grammar, it does not make predictions regarding tasks that do not rely on language, as the Y-shaped tube task described in the previous section.

Hochstein et al. (2016) investigated whether children compute ignorance inferences (10) and exclusive inferences (8). They write: "According to standard Neo-Gricean accounts, ignorance implicature and scalar implicature involve the same computations and the same formal alternatives [...] and differ only with respect to the strength of the final inference,"(p.114). Hence, such an account predicts that each child either draws both or none of these inferences. Surprisingly, they found that while 4-year-olds draw neither inference, 5-year-olds draw ignorance, but not the exclusive inference. Hochstein et al. (2016) conclude that children can reason about speaker knowledge and informativeness, and hence derive ignorance inferences, but "... fail at scalar implicature due to problems accessing relevant scalar alternative [i.e., the conjunctive alternative]"(p.132). Hence, they propose that the two inferences are somewhat independent: one is based on Gricean (epistemic) reasoning and the other on scalar (grammatical) reasoning. If we assume that children consider only one set of alternatives when interpreting a sentence, these two explanation together mean that children who do not access the scalar alternative, interpret $p \vee q$ as $p \wedge q$ (by Singh et al.) and infer ignorance about p and about q (by Hochstein et al.): $\Diamond\neg p \wedge \Diamond\neg q$. Observe that this leads to an epistemic contradiction: $(p \wedge \Diamond\neg p) \wedge (q \wedge \Diamond\neg q)$, which is an undesirable prediction of the alternative-based theory (Tieu et al., 2017). If we assume separate sets of alternatives for Gricean reasoning and scalar reasoning, nothing changes except that we allow one of the sets to be empty, allowing each inference not to be computed

[15] See the work by Aloni (2022) and Sbardolini (ms) for alternative formal ways to implement these biases. In particular, note that here we interpret information states of an agent as a snapshot of a dynamic reasoning process: it represents the information considered by that agent while interpreting one sentence, and not all the information that the agent has, as in standard theories. Hence, the complexity constraints are postulated to come from the short-term/working memory and not overall cognitive capacities. As usual in team semantics, we postulate the domain of possible worlds to be rather small and contextually determined. In particular, we do not postulate children to be completely informed about each proposition but rather that, at a given time, they can only engage with one possibility and hence are unable to form a representation more complex than what can formally be seen as a singleton information state.

despite the presence of the other, but the possibility of computing both remains open. [16]

The cognitive bias theory makes a different prediction. As we argued above, the ignorance inference arises from the basic (inclusive) meaning of disjunction together with the neglect-zero bias. Hence, we predict that conjunctive children should not draw the ignorance inference, and that children who are able to split, but still have not acquired the ability to compute scalar inferences, should draw ignorance, but not the exclusive inference. This is consistent with Hochstein et al. (2016)'s data, where 4-year-olds (known from other studies to be conjunctive) derive neither inference, but 5-year-olds (for whom conjunctivity tends to disappear) draw ignorance only.

Cochard et al. (2024) observed that while adults derive free choice inferences as in (12), some children interpret free choice sentences conjunctively as in (4). However, Singh et al. (2016) notice their parse-choice-based approach predicts that children will compute free choice inferences. On the other hand, the cognitive bias approach straightforwardly predicts that conjunctive children should also be conjunctive under a modality and derive (4). However, Cochard et al. (2024) observed that to account for this data using exhaustification, one needs to assume the implicit structure of the premise of (4) to be $\Diamond exh(Pa \vee Pb)$. They motivate such interpretation with the claim that children want to exhaustify as low as possible. This is a strong prediction that requires further empirical support, as canonically embedded implicatures are only computed in special cases (Magri, 2009, 2011).

(12) Ann is allowed to eat an apple or a banana. $\Diamond(\alpha \vee \beta)$
$\rightsquigarrow$ Ann is allowed to eat apple. $\Diamond\alpha \wedge \Diamond\beta$

(4) Ann may eat an apple or a banana. $\Diamond(\alpha \vee \beta)$
$\rightsquigarrow$ Ann may eat both an apple **and** a banana. $\Diamond(\alpha \wedge \beta)$

Bleotu et al. (2024b) show that children can be forced to access the inclusive (or exclusive) parse when faced with incompatible disjuncts as in (7). The explanation for alternative-based theories is as follows: the conjunctive alternative enters the reasoning and is negated due to world knowledge, which forces children to compute the exclusive reading. We observe that a sentence with incompatible disjuncts has exactly the same structure as a Y-shaped tube task: Since the ball cannot physically come out of both exists the *Left* and *Right* are incompatible disjuncts. We propose that the same cognitive process is involved in both the linguistic and the non-linguistic task: children who are unable to split ignore one of the possibilities entirely (Leahy and Carey, 2020).

Bleotu et al. (2025) Show that for a nonce connective (a made-up word playing a role of a connective), both children and adults tend to assign conjunctive interpretations. This data shows that conjunctive interpretations of connectives are default, and hence should be considered simpler than inclusive ones. If conjunctive behaviour on disjunctions is not due to lexical misanalysis, as we argued in 2.1.1, this data is in line with the predictions of our approach, that children default to conjunction due to its simplicity.

[16]We set aside the data by Degano et al. (2025), who show that ignorance inferences are unlikely to follow from alternative-based reasoning.

Pagliarini et al. (2018) observed that the number of conjunctive readings decreases under a universal quantifier. Instead, more children compute the distributive inference (13) or the literal (inclusive) reading. However, the exhaustification approach needs to assume that those kids have the distributive parse available as derived by Crnič et al. (2015) and that they choose it over the always available inclusive one. Interestingly, the derivation of the distributive inference also involves pruning of the conjunctive alternative; hence, it relies on sub-alternatives only without replacement from the lexicon. The difference between the conjunctive and the distributive reading is in whether children exhaustify low ($\forall x(exh(Px \vee Qx))$) or high ($exh(\forall x(Px \vee Qx))$). Hence, these results are in line with alternative-based theories. The neglect-zero approach predicts that children should draw the distributive inference only if they neglect-zero, but also perform the split.

(13) Every boy has an apple or a banana. $\forall x(Px \vee Qx)$
$\rightsquigarrow$ Some boy has an apple and some boy has a banana $\exists x Px \wedge \exists x Qx$

Other embedded environments, such as consequents of conditionals discussed in Section 2.1.1, pose a challenge to both approaches, as the rate of conjunctive interpretations varies. Alternative-based approaches can explain some of the cases away, either by appealing to the lack of implicatures in DE-environments (cf. the assumption that children exahaustify as early as possible) or assuming increasing relevance of the conjunctive alternative. The cognitive bias approach can explain the varying inference rate by the increased need or relevance of the split or by larger cognitive load of certain structures. However, the exact pattern of children's inference rates in various environments remains an open area for empirical study.

5 Conclusions

Some children interpret disjunctions conjunctively. We propose that the behaviour of these can be explained by the lack of two abilities: 1. The ability to engage with models that verify sentences by virtue of an empty witness set. 2. The ability to entertain two alternative ways the world can be in parallel (to *split*). We argue against theories explaining this behaviour in terms of alternative-based reasoning, agreeing with Chierchia et al. (2001) and Noveck (2001) that alternative-based reasoning (exhaustification) is acquired late. However, we do not claim that children are perfect logicians, but the opposite: that the incorrect conjunctive interpretations follow from cognitive biases and heuristics. In particular, our theory predicts that the order of acquisition of the meaning of disjunction starts with the conjunctive interpretation, followed by inclusive and exclusive interpretations, while exhaustification-based theories predict that children acquire the inclusive reading first. Previous research did not yield conclusive results regarding the effect of age on the acquisition of conjunctive and inclusive readings. A further empirical study based on a longitudinal design is needed to test the predictions of the two theories.

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